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Search-Space Pruning with Int-Splits

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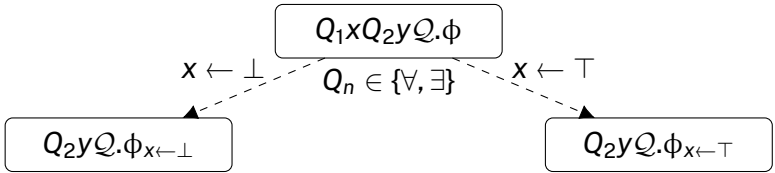
Divide and Conquer in QBF



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How deep to split?

$\log_2$ CPUs or along structures in the formula (e.g. *int-splits*)

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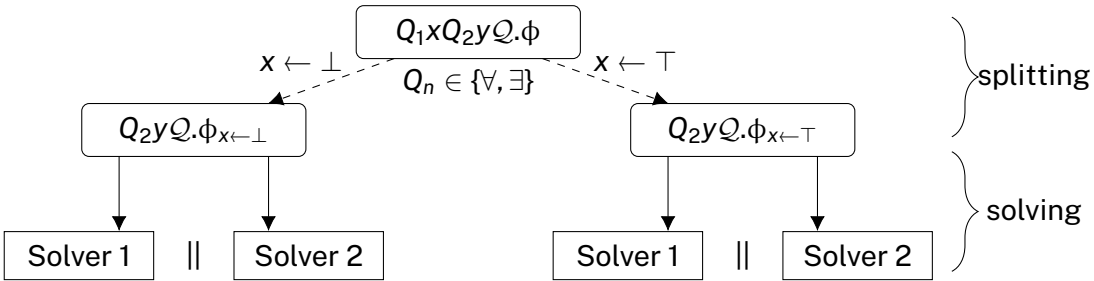
Monolithic solver architecture, the solver is one process (with sub-processes) that finishes and gives a SAT/UNSAT answer.

ParaQooba Solver Structure



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First introduced as a feature of ParaQooba *at TACAS'23, then described in another paper on Arxiv and by us here.*

Int-Splits Syntax

$Q_1(\vec{x}_1)_{c_1} \dots Q_n(\vec{x}_n)_{c_n}.\varphi$

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The constraints $c_i \in \{ <v_i,\ >v_i,\ =V,\ \top \}$ restrict the domain of the quantifier Q_i. In the definition of a constraint $v_i \in \mathbb{N}$ is a constant and V is a set of bit-vectors.

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φ is a propositional formula in CNF over the Boolean variables of the $\vec{x}_i$.

Int-Split Semantics

$\phi = Q\vec{x}_c \mathcal{Q}.\varphi$

ϕ is true iff $\nu(\phi) = 1$, with the interpretation function ν defined as follows.

$\nu(\exists \vec{x}_\top \mathcal{Q}.\varphi) = 1$ iff $\sum_{\sigma \in \mathcal{S}(\vec{x})} \nu(\mathcal{Q}.\varphi_{\sigma(\vec{x})}) \geq 1$

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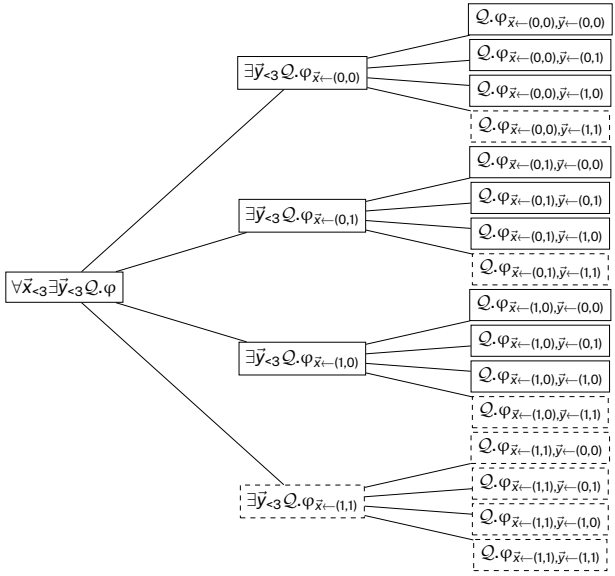
$\forall \vec{x}_{<3} \exists \vec{y}_{<3} \mathcal{Q}.\varphi$
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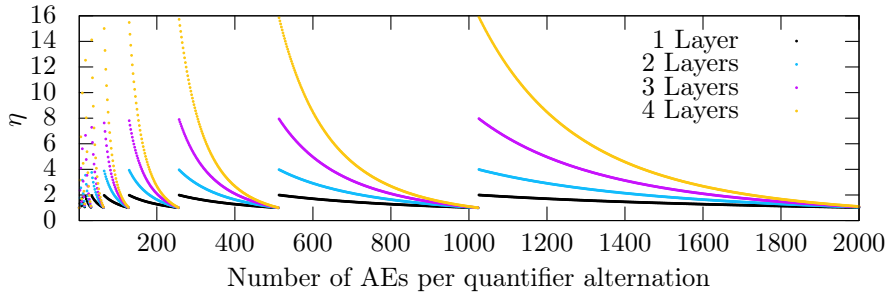
Int-Split Potential



Int-split efficiency $\eta = \frac{\text{Unaccounted Expansions}}{\text{Accounted Expansions}}$ with varying number of AEs and quantifier alternations.

QDIMACS Syntax Extension

Preamble and Int-Splits Entry-Point

⟨*input*⟩ ::= ⟨*preamble*⟩ [⟨*prefix*⟩] ⟨*matrix*⟩ EOF

⟨*preamble*⟩ ::= ⟨*comment or split*⟩* ⟨*problem line*⟩

⟨*comment or split*⟩ ::= ‘c’ ⟨*text*⟩ EOL
| ‘s’ ⟨*split*⟩ EOL *explicit change of formula semantics*

⟨*problem line*⟩ ::= ‘p cnf’ ⟨*pnum*⟩ ⟨*pnum*⟩ EOL

QDIMACS Syntax Extension

Our Int-Splits Syntax Extension

⟨*split*⟩ ::= int [⟨*int-split-vars*⟩] ⟨*int-split-list*⟩

⟨*int-split-vars*⟩ ::= ‘[’ ⟨*pnum*⟩+ ‘]’

⟨*int-split-list*⟩ ::= ⟨*int-split*⟩ [‘;’ ⟨*int-split-list*⟩]

⟨*int-split*⟩ ::= ‘<’ ⟨*pnum*⟩
| ‘>’ ⟨*pnum*⟩
| ‘=’ ‘{’ ⟨*bit pattern*⟩+ ‘}’

Selected Benchmarks with Int-Splits as Divide-and-Conquer Guide (Sequential Solver: Caqe)

Formula	#v	#c	s	u	$\#_{w/}$	η	$t_{w/}$ [h]	$t_{w/o}$ [h]	$\frac{t_{w/}}{t_{w/o}}$	$\frac{t_{seq}}{t_{par}}$
Hex 10 5x5-13	1582	5531	19	10	361	0.68	4.51	8.49	0.53	47.7
Hex 02 5x5-13	1570	5437	19	10	361	0.68	8.33	14.7	0.57	32.9
Hex 16 5x5-13	1550	5437	18	10	324	0.78	7.17	10.7	0.67	39.1
GTTT Domino	943	3075	25	10	625	0.28	0.144	0.189	0.76	11.6
GTTT El	1180	3951	25	10	625	0.28	0.265	0.320	0.83	10.4
GTTT Elly	1465	5024	25	10	625	0.28	14.6	17.4	0.84	8.59
GTTT Tic	1155	3848	25	10	625	0.28	0.612	0.726	0.84	9.47
GTTT Tippy	1409	4799	25	10	625	0.28	69.2	77.4	0.89	7.72
Hex 05 6x6-13	1812	5437	31	10	961	0.032	73.4	78.6	0.93	2.47

Ongoing: Verifying Correctness

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Conflict between what formula authors want to express and extracting hints for solvers.

Better yet: Fully commit to changed formula semantics and extend quantifiers to support int-splits?

Ongoing: Automatic Extraction & Direct Solver Support

Can we extract int-splits from existing problems? And is that even required?

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Produce solvers with int-split support and expand constraints for solvers without int-split support.

Reference Implementation of Int-Split Formula Splitter

MIT-Licensed at:

https://github.com/maximaximal/qdimacs-splitter

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